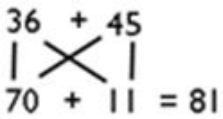
Developing fluency in addition and subtraction facts

Why the focus on fluency in addition and subtraction facts?

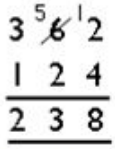
- A defined set of addition and subtraction facts built the basis of all additive calculation, just as times tables are the building blocks for all multiplicative calculation:



Informal/mental addition by partitioning:

Root addition facts

3 + 4, 6 + 5



Formal subtraction with column method

Root subtraction facts

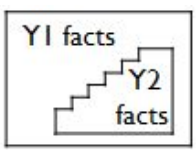
12 – 4, 5 – 2, 3 -1

- If children are not fluent in these facts, then when they are solving more complex problems the working memory is taken up by calculating basic facts, and children have less working memory to focus on solving the actual problem (See 'Is it true that some people just can't do math?' by the cognitive scientist Daniel Willingham). So fluency in basic facts allows children to tackle more complex maths more effectively.
- Fluency is one of the 3 aims of the national curriculum, and SATs for 2016 and beyond are likely to heavily test children's fluency
- Children need to be taught strategies to solve these facts. Conferencing I have done over the last year or two shows that most children don't magically become fluent in these facts even in KS2, particularly for those which bridge 10. If they aren't explicitly taught to solve e.g. 6+7 by thinking 'double 6 and one more' or to solve 12 – 8 by thinking '2 more and 2 more again' then most children will get stuck on inefficient counting based approaches.

What facts do my children need to be fluent in?

The full set of addition facts is here:

+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	3+10
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	4+10
5	5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10
6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	7+10
8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10	10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10



And here are the corresponding subtraction facts.

-	0	1	2	3	4	5	6	7	8	9	10
1	1–0	1–1									
2	2–0	2–1	2–2								
3	3–0	3–1	3–2	3–3							
4	4–0	4–1	4–2	4–3	4–4						
5	5–0	5–1	5–2	5–3	5–4	5–5					
6	6–0	6–1	6–2	6–3	6–4	6–5	6–6				
7	7–0	7–1	7–2	7–3	7–4	7–5	7–6	7–7			
8	8–0	8–1	8–2	8–3	8–4	8–5	8–6	8–7	8–8		
9	9–0	9–1	9–2	9–3	9–4	9–5	9–6	9–7	9–8	9–9	
10	10–0	10–1	10–2	10–3	10–4	10–5	10–6	10–7	10–8	10–9	10–10

Note that in subtraction facts not all subtractions within 20 are root facts, e.g. 17-5 is not considered a root fact (7 -5 is the root fact for this.)

The majority of these facts will be learnt in Ys1&2.

In reception, children become fluent in working with totals to 5 (though not recording as number sentences), e.g. "Show me 5 on your hands. Now show me 5 in a different way."

Year 3 will need to focus on securing fluency in subtraction facts which bridge 10. Although this is a Year 2 objective, my feeling after many hours teaching and reflecting on factual fluency is that aiming for real fluency in subtraction facts such as 14 – 9 and 13 – 5 (where fluency is an answer in 3 seconds) for each and every child in Y2 is unrealistic. Unless we are honest about that and accept the need to secure this in Year 3, we risk having children who never become secure in this,

<u>Does fluency just mean memorisation?</u>

Not necessarily – when you conference adults on how they solve addition and subtraction facts, almost all adults rely on very quick use of strategies to solve some of them. Reflect carefully on the set of addition and subtraction facts shown: which have you memorised and which are you very quickly deriving? I've taken fluency to mean 'getting an answer pretty quickly and with limited demands on working memory'. Aim for an average of 3 seconds or less per fact. My work conferencing fluent children in KS2 who were working at this speed showed:

- Most facts which didn't bridge 10 were memorised – the children reported 'just knowing' than 4 + 5 = 9 or 2 + 6 = 8 for example.
- For facts which bridge 10, the picture is more complex and many of the facts which bridge 10 were quickly derived using strategies (but still in less than 3 seconds!).
    - Double 6, 7. 8 and 9 were always memorised in fluent children
    - Many fluent children also reported 'just knowing' that 9 + 3 = 12 and 8 + 4 = 12 and related this to their times table/skip counting knowledge.
    - Fluent children in the year groups conferenced (up to Year 4) generally reported using strategies for many of the other facts. 9 + 8 is an example of a fact that actually very few people (either adults or children) have memorised. Most fluent people solve this through very quickly applying a strategy: bridging, near doubles or compensating.

As a reference point, the grid below is a good example of the approaches taken by a fluent, high attaining Year 4 child to each of the addition facts: he doesn't use a counting approach for any of the facts, but he has certainly not memorised them all either (K= Known fact; S= Strategy)

	0	1	2	3	4	5	6	7	8	9	10
0	K	K	K	K	K	K	K	K	K	K	K
1	K	K	K	K	K	K	K	K	K	K	K
2	K	K	K	K	K	K	K	K	K	K	K
3	K	K	K	K	K	K	K	K	S	S	K
4	K	K	K	K	K	K	K	S	K	S	K
5	K	K	K	K	K	K	S	S	S	S	K
6	K	K	K	K	K	S	K	S	S	S	K
7	K	K	K	K	S	S	S	K	S	S	K
8	K	K	K	S	K	S	S	S	K	S	K
9	K	K	K	S	S	S	S	S	S	K	K
10	K	K	K	K	K	K	K	K	K	K	K

Name	MS
School	AD
Year	4
Level	3a
Notes on strats	
Predominantly bridging	
Diff of 2 converted to doubles e.g. 6 + 8 saw as double 7	
Strong commutativity - identical strats always used	

Please can everyone find the time to take one of your middle attaining children and see how they solve each of the 121 facts. They are all written out on the final page of this memo – just print and cut them out. I use these sorting circles with the children, I have found they very quickly get the idea – if they say they would use counting don't bother getting them to solve the fact – you will be there all day! Just identify their approach. If they say strategy I find it interesting/helpful to ask what strategy they have used (see notes next to child MS's grid above).

How do children become fluent?

As mentioned above, children need to be **TAUGHT** strategies to derive the facts! An interesting piece of research (Thornton, 1976) showed that teaching strategies is more effective in securing fluency in addition and subtraction facts than taking a rote memorisation approach. That is to say, even if your aim is memorisation, the most effective way to get there is through the teaching of strategies. There is a huge amount to unpick in this and you need to consider how children are going to become fluent in each and every fact. For example, we want children to just know that 4 + 2 = 6 and 9 – 2 = 7 etc so we need to teach children that when we add 2 or subtract 2 we are moving to the next/previous even number (if starting on an even) or odd number (if starting on an odd). Without being taught this, many children will count, e.g. nine: eight, **seven** for 9 – 2.

The separate document 'progression in teaching addition facts' lays out a suggested progression and teaching approaches for each stage of this progression:

1. Adding 1 (e.g. 7 + 1 *and* 1 + 7)
2. Doubles of numbers to 5 (e.g. 4 + 4)
3. Adding 2 (e.g. 4 + 2 *and* 2 + 4)
4. Number bonds to 10 (e.g. 8 + 2 *and* 2 + 8)
5. Adding 0 to a number (e.g. 3 + 0 *and* 0 + 3)
6. Adding 10 to a number (e.g. 5 + 10 *and* 10 + 5)
7. The ones without a family! 5 + 3, 3 + 5, 6 + 3, 3 + 6 (these last two can be related to counting in 3s)
8. Doubles of numbers to 10 (e.g. 7 + 7)
9. Near doubles (e.g. 5 + 6 *and* 6 + 5)
10. Bridging (e.g. 8 + 4 *and* 4 + 8)
11. Compensating

Note that these 3 strategies can often be used interchangeably, e.g. for 8 + 9, some people will use near doubles (e.g. 8 + 8 + 1), some will use bridging (e.g. 8 + 2 + 7) and some will use compensating (8 + 10 – 1)

N.B. Before the children are ready to learn bridging as a strategy, they need to be able to partition all single digit numbers. See separate doc for an approach for this.

There is also a separate smartboard file called 'developing fluency in subtraction facts' which will support you with teaching just that. There is even more pedagogy involved in supporting children to become fluent in subtraction facts then there is in addition facts!

Once children have been taught the strategies, they need to move on to **PRACTICE** of the facts, Remember for many facts the ultimate aim of the practice is memorisation, for others the aim of the practice is increasing speed and fluency in the applied strategy. In all cases we are aiming for an average of 3 seconds or less per fact. This means that in 2 minute practice session the children should be recalling at least 40 facts: the more you practice the quicker they get, and the quicker they get the less time it takes out of the lesson!

Generally for practice

- Focus on practising the set of facts being learnt (or just learnt) in isolation for a few days
- The focus on mixing these up with all previously learnt facts

Ways that you can practice might include

- Number gym software: children can also access this at home and you can access their times through it
- Class flash cards of the facts that we are learning or the facts that we know already. Have these written on A4 paper and practice them in spare seconds here and there: lining up for assembly, coming in after break etc. {I'd tend to review the strategy for each too, e.g 'that"s right, double 6 add 1' after looking at 6 + 7}
- Speed sheet tests for children. Once they get fluent in finding totals or differences quickly, you can challenge them further by including missing addend or minuend questions.

Good luck with it and as always just ask if any qus!

Addition facts: cut these up for pupil conferencing

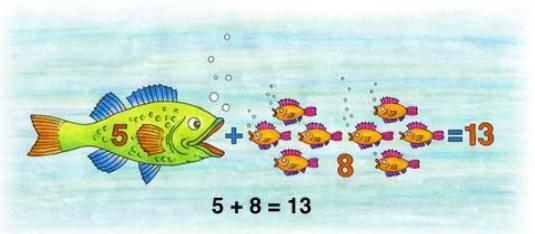
A note about those with gaps

0 + 0	0 + 1	0 + 2	0 + 3	0 + 4	0 + 5	0 + 6	0 + 7	0 + 8	0 + 9	0 + 10
1 + 0	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	1 + 7	1 + 8	1 + 9	1 + 10
2 + 0	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3 + 0	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4 + 0	4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5 + 0	5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6 + 0	6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7 + 0	7 + 1	7 + 2	7 + 3	7 + 4	7 + 5	7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8 + 0	8 + 1	8 + 2	8 + 3	8 + 4	8 + 5	8 + 6	8 + 7	8 + 8	8 + 9	8 + 10
9 + 0	9 + 1	9 + 2	9 + 3	9 + 4	9 + 5	9 + 6	9 + 7	9 + 8	9 + 9	9 + 10
10 + 0	10 + 1	10 + 2	10 + 3	10 + 4	10 + 5	10 + 6	10 + 7	10 + 8	10 + 9	10 + 10

+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	3+10
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	4+10
5	5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10
6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	7+10
8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10	10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10

For a few children the cognitive load in bridging seems to be enormous. For those who gaps need to be plugged quickly here is an approach for facts which bridge 10 without the need for bridging.

- Doubles and near doubles – gives the grey and orange Y2 facts.
- Revisit adding 10, and use this to move to adding 9 using compensating.
- This leaves just 6 facts to learn (plus the commutatives)
    - 7 + 4 (one more than 7 + 3)
    - 7 + 5 (5 + 5 and two more)
    - 8 + 3 (one more than 8 + 2)
    - 8 + 4 (relate to counting in 4s)
    - 8 + 5



    -
    - 8 + 6