

Secondary Teaching for Mastery Self Evaluation

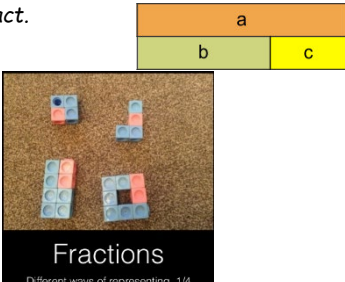
Your name		School & Role		Date	
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The aim of this document to give schools and departments a guide to implementing features of teaching for mastery across the mathematics curriculum. It is based on the NCETM documentation and the exemplification of mastery is based on what has been seen in Shanghai lessons and in textbooks from China, Singapore and Japan.

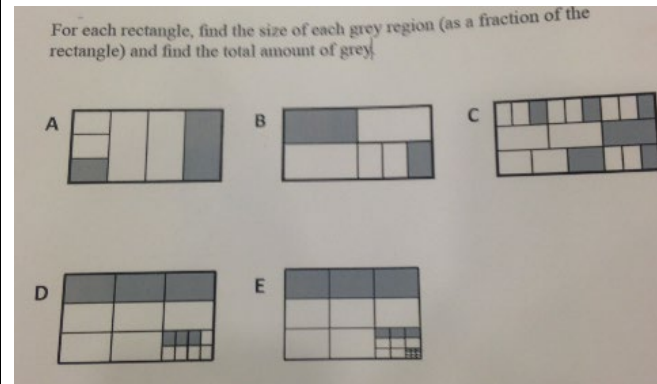
0: Isn't something we do currently 1: Sometimes happens/starting to think about this 2: Happens fairly often but not embedded 3: Is a central feature of our practice

Principles and Beliefs	Feature of Mastery	Exemplification/further detail	0	1	2	3
	The ethos throughout the school is that the essential idea behind teaching for mastery in mathematics is that all students need a deep understanding of the mathematics they are learning so that future mathematical learning is built on solid foundations which not need to be retaught	<i>The school recognises that to develop confident mathematicians solid foundations are the key. 'Progress' through key concepts may be perceived as much slower than current practice but as a deep conceptual understanding is developed future learning may be more 'rapid'. The idea of 'rapid progress' is seen very much over the longer term.</i>				
	All adults proactively promote a 'can do' attitude to mathematics for all students. All students are encouraged to develop a growth mind-set	<i>The messages that adults give to students have a big impact on their thinking. All adults encourage students to 'have a go' and to celebrate wrong answers as a learning opportunity. Using the terminology 'YET' can be very powerful. 'I haven't got this yet, I need more practise'. 'You can do this you just not yet, I will help you to know it.'</i>				
	All adults do not label students such as 'good/no good at maths' and 'high/low ability' (based on previous attainment). All adults believe that the vast of majority of students can attain mastery of the key ideas in mathematics.	<i>No pupil is prejudged due to prior attainment. Each new phase is a new start, each new day is a new start. Success comes from hard work and effort not prior academic labels. Students experience the same curriculum.</i>				
	All adults believe success is linked to effort and hard work, they celebrate and praise the thinking behind students' answers rather than the 'correct' answer	<i>Building resilience in to students is very important. Adults must develop deep thinking skills and resilience in students through praising the application of a process rather than always when the correct answer is given. The answer is just the beginning.</i>				
	All adults understand mastery of mathematics is not a fixed state but a continuum	<i>Mathematics is seen as a journey and each stage and topic is a step along the path or bricks in a wall. The more steps or bricks students lay the more they can achieve but we must not miss any steps out nor create unsteady foundations.</i>				
	The class work together on the same key point, whilst at the same time challenging and supporting students to gain depth of understanding and proficiency. Acceleration to higher content is avoided.	<i>By starting every pupil at the same point and by designing lessons so carefully that errors can be avoided gaps in students understanding can be minimised. Each step is so small there is almost errorless learning. Rapid graspers are challenged through carefully selected questions which deepen their understanding and ability to reason their answers.</i>				

	<p>The school recognises that very successful lessons come from expert teachers delivering lessons which have been designed together, reviewed and improved over time.</p>	<p>Wherever possible classes are taught by expert teachers and there is an acknowledgement that teaching by non-experts, particularly in lower years can lead to misconceptions later.</p> <p>The school ensures that all meetings are focused on teaching and learning and not administration or data. Group planning becomes the norm.</p> <p>The school supports regular lesson study between colleagues and between local groups of schools. CPD is a high priority and is focused on long term deep conceptual understanding rather than short term interventions.</p>				
	<p>Any comments on Principles and Beliefs:</p>					
Curriculum Design	<p>The school has evaluated the structure of the curriculum and designed the lessons so that there is a suitable amount of time allocated to mathematics.</p>	<p>Schools may decide that little and often is preferable to larger blocks of time. One lesson of mathematics every day in the morning may suit many students. Setting or not of students by ability is carefully considered, particularly in lower years.</p>				
	<p>A detailed curriculum is mapped out across all years, ensuring continuity and supporting transition. Maths is taught in small, carefully sequenced step which build conceptual understanding of the basic skills so that students can succeed in later topics.</p>	<p>Teachers know exactly what has been taught in previous year groups, how this feeds into their year group, and how it will be built on in the subsequent year group. Transition from previous schools and phases is carefully considered and the school actively seeks out information from feeder schools. Student voice is used to improve transition. Regular opportunities are developed in all years to practise fundamental skills.</p>				
	<p>We ensure that children master each step before moving to the next stage.</p>	<p>There are clear minimum written expectations for each year that all students (except those on individualised curricula) will leave the year group having mastered. Examples may be provided by the school in a departmental portfolio. It is not expected that concepts and procedures will be retaught later in the year or the following year; instead they will be built on.</p>				
	<p>Considerable time is spent on securing fundamental skills, knowledge and vocabulary in the early stages.</p>	<p>A unit on proportion would cover fundamental ideas such as ratio, fractions and proportion. Teachers would be clear of the similarity and differences between these and would use them correctly, especially when using multiplicative reasoning. Students would practise telling proportion stories, and e.g. might explore the fraction $\frac{1}{4}$, the ration 1:3 and the proportion of one in four or 25%. They would work only within a carefully defined number set, the whole being 60 for example, and aim to become fluent in these fundamental relationships.</p> <p>Students understand the concept of the distributive laws and the commutative laws and how they can be used flexibly to aid written and mental calculations.</p>				
	<p>Any comments on curriculum design:</p>					

Lesson Design & Resources	<p>Concrete and pictorial representations are chosen carefully to help build procedural and conceptual knowledge together, leading to the ability to apply in abstract situations.</p>	<p>Teachers carefully choose the 'best' models to explain a concept, rather than putting all of the resources out and letting the child choose what to use. Teachers can explain how using representations will help the student <u>understand</u> the maths rather than just <u>do</u> the maths. Because the modelling methods and tools have been chosen to help build understanding of the concepts, they allow students to move to working in the abstract. Teachers use the 'Bar Model' consistently across all years.</p> <p>e.g. Use multilink cubes to reinforce the idea of fractions.</p> 																				
	<p>Exercises are structured with great care to build deep conceptual knowledge alongside developing procedural fluency</p>	<p>Variation theory in practice.</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td>Set A</td> <td>Set B</td> <td></td> </tr> <tr> <td>120 – 90</td> <td>120 – 90</td> <td rowspan="6" style="font-size: small; vertical-align: middle;">Example of Variation by Mike Askew 'Transforming Primary Mathematics'</td> </tr> <tr> <td>235 – 180</td> <td>122 – 92</td> </tr> <tr> <td>502 – 367</td> <td>119 – 89</td> </tr> <tr> <td>122 – 92</td> <td>235 – 180</td> </tr> <tr> <td>119 – 89</td> <td>237 – 182</td> </tr> <tr> <td>237 – 182</td> <td>502 – 367</td> </tr> </table> <p>Compare the two sets of calculations below. What's the same, what's different? Which of the two would you use? Consider how variation can broaden or narrow the focus. Frequent practice like in Set B helps patterns to become ingrained. Students will be able to see that question B2 is very similar to B1 and hence reduce the need to calculate formally. Students are explicitly taught to investigate questions like $34 + 78 + 66$ to see that by changing the question it becomes $34+66+78 = 100 + 78 = 178$. These flexible mental strategies are practised and become second nature to students.</p>	Set A	Set B		120 – 90	120 – 90	Example of Variation by Mike Askew 'Transforming Primary Mathematics'	235 – 180	122 – 92	502 – 367	119 – 89	122 – 92	235 – 180	119 – 89	237 – 182	237 – 182	502 – 367				
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<p>Teachers have access to high quality resources to support lesson planning (e.g. text books, schemes of work)</p>	<p>Any printed resources being used have the aim of developing <u>teacher pedagogical subject knowledge</u> at their heart. They are mathematically coherent, highlight common misconceptions and show clear and appropriate representations.</p>																					
<p>Lessons are crafted with care, are discussed with other teachers, and draw on evidence from observations of students in class.</p>	<p>Teachers have a clear focus on what students will learn in the lesson, not just what they will <u>do</u> in the lesson. There may be explicit different lesson types for different parts of a unit. 'Concept' lesson, 'Knowledge' lesson, 'Calculation' lesson, 'Practice' Lesson. There may be different expectations of the amount of formal written work in the different types of lesson. In a 'Concept' lesson all calculations may be done by calculator as not to let errors in arithmetic interfere with new ideas.</p> <p>Learning intentions are very tightly focused and much narrower than traditional learning objectives. A learning objective of 'To be able to add proper fractions' may be split into its component parts and taught over a series of lessons. As part of lesson planning, teachers reflect with colleagues on what they have noticed about students' learning in class. They talk to each other about 'key difficulty points'/misconceptions/hinge questions and how best to present these ideas.</p>																					

There is a focus on developing thinking and reasoning skills and a realisation that the 'answer' is just the beginning of a discussion and not the ultimate goal.



Lesson designs set out in detail well tested methods to teach a given mathematical topic

Once teachers have identified 'key difficulty points'/misconceptions/hinge questions, they plan to address these carefully in teaching. For example, they may know that children sometimes think a shape divided into 4 is always divided into quarters, and so would teach 'divided equally' and 'not divided equally' (the concept and the non-concept). They would look at images of each & identify that only those 'divided equally' into 4 are quartered. Similarly, with the common confusion between Ax^b & $(Ax)^b$.
Teachers stick to the concrete, visual, abstract ideas for lesson design.

Teachers include a variety of representations to introduce and explore a concept effectively

In the above example, teachers would explore several different ways of 'dividing equally into 4', starting with shapes (e.g. divided across, down, diagonally). They would then extend to look at equally dividing sets of objects into 4, equally dividing continuous quantities into 4 (e.g. liquid) to explore the concept in depth. With the common confusion between Ax^b & $(Ax)^b$, these can be illustrated with both substitution and graphically. This can lead to depth depending on the values of constants A and b.

Teachers set out related teacher explanations and questions to students

Teachers have identified what the potential misconceptions are, and carefully plan 'hinge questions' to test for these in children.
Calculate $\frac{1}{2} + \frac{1}{3}$ A) 2/5 B) 2/6 C) 5/6 D) 5/12

Any comments on Lesson Design & Resources:

Classroom Practice	Teachers are clear that their role is to teach with precision, which makes it possible for all students to engage successfully with tasks at the expected level.	Teachers use precise pre planned language (e.g. 'this is the whole, this is the part of the whole, quotient, numerator and denominator' in a lesson on fractions), have fully thought through the models and explanations they will use, and believe that this will enable all children to develop as mathematical thinkers and so succeed. Teachers reinforce fundamental laws regularly and effectively using precise vocabulary 'commutative', 'distributive', 'dividend'				
	Students work on the same tasks and engage in common discussions.	Differentiation is about depth rather than acceleration. Teachers may differentiate by the way they allow students to use concrete manipulatives or by providing a different explanation for some students. The expectation is that all students study and can learn the same material. Rapid graspers are challenged by tackling deeper questions rather than new topics e.g. applying Pythagoras' theorem to practical problems or in 3D.				
	Concepts are often explored together to make mathematical relationships explicit and strengthen students understanding of mathematical connectivity	For example, in lessons, the teacher and children will highlight several different approaches to solve the same calculation (e.g. $198 + 997$ could be solved by partitioning, using column addition, or by $200 + 1000 - 5$, or by 'moving a 3' to give $195 + 1000$ etc) which would be discussed as a whole class. The Chinese call this ' <u>active argument</u> '. Let us take this problem. What value of x will make the statement $2(x-5) = 20$ true?. The strategy is to express the terms in equivalent forms. $2(x-5) = 20$ can be expressed as $2(x-5) = 2(10)$. $2(x-5) = 2(10)$ implies $(x-5) = 10$ $x-5 = 10$ can be expressed as $x-5 = 15-5$. Thus $x = 15$. This way of thinking can be used to solve the equation $2(x+7) = 4x$. $2(x+7) = 2(2x)$ $\Rightarrow (x+7) = 2x$ $\Rightarrow x+7 = x+x$ $\Rightarrow x = 7$.				
	Precise questioning during lessons ensures that students develop fluent technical proficiency and think deeply about the underpinning mathematical concepts.	Teachers plan questions to build depth as well as fluency, e.g. how can we use the idea of 'same difference' in subtraction to solve $82 - 64 = 78 - \underline{\quad}$ without having to calculate that $82 - 64 = 18$				
	Any comments on teaching methods:					

Support & Differentiation

Taking a mastery approach, differentiation occurs in the support and intervention provided to different students, not in the topics taught, particularly at earlier stages. There is no differentiation in content taught, but the questioning and scaffolding individual students receive in class as they work through problems will differ. Higher attainers are challenged through more demanding problems which deepen their knowledge of the same content rather than being moved onto content from future year groups.

A KS3 student, after learning to add proper fractions may be asked to explain or draw visualisations for $(ad+bc)/bd$ rather than progress on to mixed numbers. Which calculation is the odd one out?

- 753×1.8
- $(75-3 \times 3) \times 6$
- $753 + 753 \div 5 \times 4$
- 7.53×1800
- $753 \times 2 - 753 \times 0.2$
- $750 \times 1.8 + 3 \times 1.8$

Explain your reasoning.

Only a fraction of each whole rod is shown. Using the given information, identify which whole rod is longer.



Explain your reasoning.

A KS4 student, after learning to sketch quadratic equations may be asked to express the same quadratic equation in three different forms and spot the connections between the different forms and the graphs.

Students' difficulties and misconceptions are identified through immediate formative assessment and addressed with rapid intervention - commonly through individual or small group support later the same day: there are very few "closing the gap" strategies, because there are very few gaps to close.

Teachers have time the same day to take students out of class and provide additional teaching for any children who had difficulties in the lesson and who would otherwise fall behind.

Any comments on pupil support and differentiation:

Productivity & Practice	<p>Fluency comes from deep knowledge and practice. Students work hard and are productive. At early stages, explicit learning of multiplication tables is important in the journey towards fluency and contributes to quick and efficient mental calculation. Practice leads to other number facts becoming second nature.</p>	<p>Students have excellent knowledge of number facts due to explicit teaching of them. For example, children will all have been taught, and practised, operations with fractions, decimals and percentages before being taught probability (i.e. facts such as $1/5 \times 1/3$, $0.25 + 0.4$ on which mastery of probability is partly dependent). Practice of facts may include immediate application e.g. $6 \times 5 = 30$, $6 \times 5p = 30p$, $6 \times 5kg = 30kg$.</p>																			
	<p>The ability to recall facts from long term memory and manipulate them to work out other facts is also important.</p>	<p>One aim is fluency in all addition facts, and linkages between facts are used to achieve this. In learning addition facts, children might be taught to see that $35 + 19$ is one less than $35 + 20$ to help them become fluent in the fact $35 + 19 = 54$. Students used multiplication facts flexibly so that they 'see' that $14 \times 12 = 10 \times 12 + 4 \times 12$ rather than revert automatically to a written method.</p>																			
	<p>All tasks are chosen and sequenced carefully, offering appropriate variation in order to reveal the underlying mathematical structure to students. Both class work and homework provide this 'intelligent practice', which helps to develop deep and sustainable knowledge.</p>	<p>Consider the construction of this task.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>$2 \times 3 =$</td> <td>$6 \times 7 =$</td> <td>$9 \times 8 =$</td> </tr> <tr> <td>$2 \times 30 =$</td> <td>$6 \times 70 =$</td> <td>$9 \times 80 =$</td> </tr> <tr> <td>$2 \times 300 =$</td> <td>$6 \times 700 =$</td> <td>$9 \times 800 =$</td> </tr> <tr> <td>$20 \times 3 =$</td> <td>$60 \times 7 =$</td> <td>$90 \times 8 =$</td> </tr> <tr> <td>$200 \times 3 =$</td> <td>$600 \times 7 =$</td> <td>$900 \times 8 =$</td> </tr> </table> <p>Why does this demonstrate 'intelligent practice'? What mathematical structures are being developed/embedded?</p>	$2 \times 3 =$	$6 \times 7 =$	$9 \times 8 =$	$2 \times 30 =$	$6 \times 70 =$	$9 \times 80 =$	$2 \times 300 =$	$6 \times 700 =$	$9 \times 800 =$	$20 \times 3 =$	$60 \times 7 =$	$90 \times 8 =$	$200 \times 3 =$	$600 \times 7 =$	$900 \times 8 =$				
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<p>Success is linked to hard work, and there is a belief that all can succeed in maths.</p>	<p>Teachers focus praise on process "Well done, you have used a fact you knew already to work out a new fact" rather than e.g. "Wow, you worked out the answer really fast. You are so good at maths". "That's very interesting I am glad you said that"</p>																				
<p>Any comments on productivity and practice:</p>																					

Assessment	Assessment values knowing 'why' as well as knowing 'that' and knowing 'how'	<i>Assessments may ask for students to visualise problems as well as obtain a final solution.</i>				
	Assessment does not solely focus on the need to memorise key facts and procedures and answer test questions accurately and quickly	<i>Questions are designed carefully so that students can use number facts flexibly to answer problems. Questions are not only 'do' questions. Questions may contain information not required for the problem so ensure that students develop the ability to decode questions confidently.</i>				
	Assessment values applying mathematics to new and unfamiliar situations	<i>Assessments focus in using current learning in a variety of different situations to ensure that concepts are understood conceptually rather than procedurally.</i>				
	Both class work and homework support and develop 'intelligent practice', which helps to develop deep and sustainable knowledge	<i>Classwork and homework carefully reflect the needs of the students. Classwork and homework is not set rigidly to a scheme of work but builds on quality assessment for learning.</i>				
	Fluency comes from deep knowledge and practice	<i>Regular assessment of the fundamental concepts is performed ensuring there is sufficient time in the curriculum to recall prior learning.</i>				
	Assessment reflects not only written work but students' explanations.	<i>Ongoing assessment is crucial for teachers to ask probing questions that deepen understanding. Formal written assessments only form a part of the overall picture of a student's understanding.</i>				
	Assessment reflects the expectations of the year group and not always a final end of key stage forecast.	<i>Grading reflects the building of knowledge rather than a current assessment of knowledge. Lower years are graded to reflect their potential.</i>				
	<i>Any comments on assessment:</i>					