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Primary & Early Years Magazine 77

New National Curriculum in Focus

New National Curriculum in Focus** is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study for mathematics. You can find previous features in this series **here

Designing learning for short division

The new curriculum requires children to learn to use standard written methods sooner than has been taught in recent years. In their 2011 report **Good practice in primary mathematics: evidence from 20 successful schools**, Ofsted identified that in the most successful schools pupils were moved to standard written methods swiftly, and once pupils were secure with interim methods, they were moved quickly on to more efficient methods.

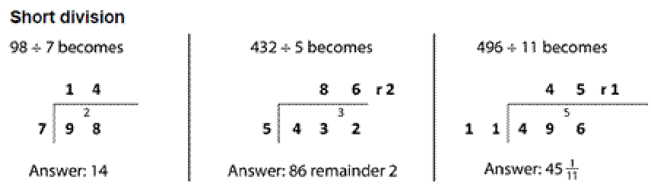
What is important is that, if pupils are to be expected to move to a standard written method more quickly than previously expected, we need to ensure that that they do this not only with procedural fluency but with conceptual understanding.

So can this be achieved for short division?

Let's consider the Y6 statement from the programme of study:

> *divide numbers up to four digits by a one-digit number using the formal written method*

Appendix 1 of the National Curriculum suggests this method for short division:



Before pupils can begin to learn to do this there are a number of skills and concepts that need to have been developed in order to carry out short division:

- recall fluently multiplication facts to 12 x 12, recognise multiples
- visualise and understand how a four-digit number can be partitioned and recombined into multiples of 1000, 100, 10 and 1 with both concrete and abstract representations. (i.e. base 10 (concrete), place value counters or arrow cards)
- visualise the relative quantity of the numbers
- know the value of a digit because of its position in a number
- understand the effect of multiplying by 10, 100 and 1000
- understand that multiplication and division are inverses and use this relationship to estimate and check answers
- decide when it is more efficient to calculate mentally
- understand the concept of a remainder after division
- understand that division is (left) distributive over addition, eg (a + b) ÷ c = (a ÷ c) + (b ÷ c)

Scaffolding learning through procedural and conceptual variation

One particular feature of the teaching seen in Shanghai has been the use of teaching with conceptual and procedural variation. You can read in more detail what this means in National Curriculum in Focus from **Issue 73**.

So what might conceptual and procedural variation look like in the context of teaching short division of four-digit numbers by a single-digit number?

An effective representation of multiplication is to consider this as an array.

e.g. Looking at the columns we can see 7 x 14 = (7 x 10) + (7 x 4), but also reveals from the rows that 14 x 7 = 7 x 14 = 98:

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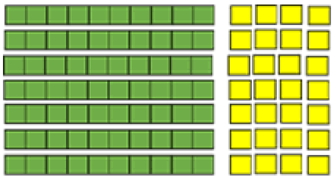
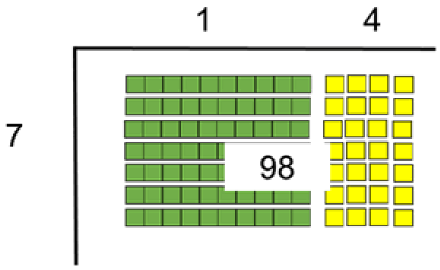


This can help to understand the abstract notation of often referred to as the ‘bus stop’ method:



There is one lot of seven 10s and four lots of seven ones.

So how is this translated to a written method for division?

Firstly, in choosing examples, we should be mindful of the different skills that need to be developed. In previous articles we have separated out examples that require regrouping from those that do not. So when dividing a 4 digit number by a single digit number using short division, we should begin with examples that do not require regrouping to develop a secure method.

Let’s consider the problem: ***A biscuit factory makes the same number of biscuits every hour. If it makes 3636 biscuits in three hours, how many biscuits are made every hour?***

By Y6 we should expect pupils to recognise that this is a division problem. By acting out the problem with concrete resources first this will enable any uncertain pupils that the actions they are performing related to the abstract problem 3636 ÷ 3 = ?

Using Dienes can become cumbersome as numbers become larger so the use of place value counters can be helpful alternative concrete abstraction to illustrate the process.

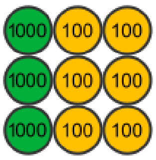
Building 3636 with place value counters will present this:

How many times can we share three thousand between three?

What’s the same, what’s different?

	Th	H	T	1s
	1			
3	3	6	3	6

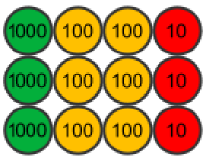
How many times can we share six hundred between three?



What’s the same, what’s different?

	Th	H	T	1s
	1	2		
3	3	6	3	6

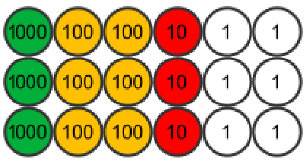
How many times can we share thirty (or three tens) between three?



What’s the same, what’s different?

	Th	H	T	1s
	1	2	1	
3	3	6	3	6

How many times can we share six (or six ones) between three?



What’s the same, what’s different?

	Th	H	T	1s
	1	2	1	2
3	3	6	3	6

The variations from left to right draw the pupils’ attention to the meaning of the abstract representation. Opportunities should be given for pupils to articulate these similarities and differences by posing questions that help to reveal the pupils’ deepening understanding. For example – why do you think these are different? Why do you think these are the same? Pupils should be encouraged to seek the similarities and differences and at each successive step compare what has changed and what has remained the same (see Issues **75**, **74** and **73** for further examples in the context of other operations).

Vary the numbers slightly in the same word problem. For example this sequence will enable pupils to notice slight variations in each problem in terms of the recording:

3639 ÷ 3 =

3669 ÷ 3 =
3969 ÷ 3 =
6969 ÷ 3 =

All of the above problems require simple division and no regrouping when dividing by three. Pupils should then have the opportunity to practise for fluency in the context of problems with other divisors, dividends and contexts, moving away from the need to use the concrete resources when pupils are confident that they understand the method.

Pupils' understanding could be deepened by asking them to finding possible answers for the numbers hidden by the stars:

	Th	H	T	1s
	1	2	1	4
★	★	★	★	★

However another problem such as 3642 ÷ 3 will then enable a shift in the children's emerging thinking and generalising of the procedure to situations where regrouping is required.

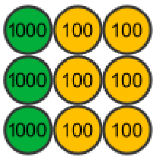
As before, build the number with place value counters:

How many times can we share three thousand between three?

What's the same, what's different?

	Th	H	T	1s
	1			
3	3	6	4	2

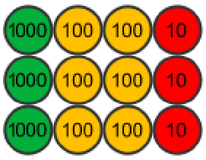
How many times can we share six hundred between three?



What's the same, what's different?

	Th	H	T	1s
	1	2		
3	3	6	4	2

How many times can we share four tens between three?



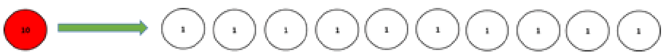
What's the same, what's different?

	Th	H	T	1s
	1	2	1	
3	3	6	4	^{1}2

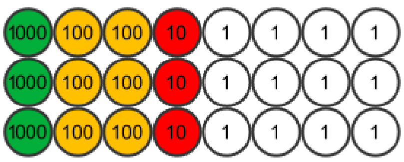
At this point the children will discover that they can share four tens between three only once, and that there is a **remaining** ten (and two ones) to deal with (referring to this as a 'remaining ten' will enable connections from earlier or later with the term 'remainder'). This can be described as a 'pivot point' for their learning because the procedure they have become familiar with without regrouping is now being challenged, enabling an opportunity to deepen their understanding.

Invite pupils to discuss what can be done next. One suggestion might be to place the **remaining** ten in the next column and finish the column off with the remaining ones. It will therefore be important to have noticed and mentioned in previous examples the fact that in each row of every column the value of each counter is the same

Experience from previous examples in other operations and previous experience in short division should enable some pupils to suggest regrouping the one ten into ten ones:



After regrouping the remaining ten we can ask: *How many times can we share twelve (ones) between three?*



What's the same, what's different?

	Th	H	T	1s
	1	2	1	4
3	3	6	4	^{1}2

Comparing the concrete representation with the abstract notation will be an interesting discussion point by asking *Where is the remaining ten that we regrouped into ones?*

Provide further sequences of examples where one ten requires regrouping into ten ones and withdrawing from the concrete resources as pupils develop confidence in recording the procedure, e.g.

3642 ÷ 3
3645 ÷ 3
3648 ÷ 3

4852 ÷ 4
4856 ÷ 4

7784 ÷ 7

8896 ÷ 8

At this point you may wish to ask pupils to observe any emerging patterns – in the above the thousands and hundreds digits are multiples of the divisor and the tens digit is 'one more' than the divisor. To deepen understanding, ask pupils to suggest another where the tens will need to be regrouped into ten ones.

Pupils could then be asked to conjecture what will happen if instead of the problem being 3642 ÷ 3 the problem is 3651 ÷ 3. At this point there would be *two* tens that need regrouping into 20 ones. This might be cumbersome with the resources on the table so demonstrating the process with the interactive whiteboard may be more helpful. This will then allow a discussion to evaluate how efficient it is using the resources compared with using just the written notation.

Compare the abstract notation to make sense of the procedure:

	Th	H	T	1s
	1	2	1	4
3	3	6	4	$^{1}2$

What's stayed the same? What's changed?

	Th	H	T	1s
	1	2	1	7
3	3	6	5	$^{2}1$

Provide a sequence of problems for pupils to work through that involves regrouping two tens and then other multiples of tens for different divisors. Again ask pupils to identify any patterns to help them predict whether any regrouping is required.

Other 'pivot' points to include in teaching of short division:

- dividing a four-digit number by a single-digit number where the hundreds require regrouping
- dividing a four-digit number by a single-digit number where the thousands require regrouping
- dividing a four-digit number by a single-digit number where the tens and hundreds, tens and thousands or hundreds and thousands require regrouping
- dividing a four-digit number by a single-digit number where all digits require regrouping
- dividing a four-digit number by a single-digit number where one digit is zero
- dividing a four-digit number by a single-digit number where two or three digits are zero
- dividing a four-digit number by a single-digit number where the thousands, hundreds or tens is less than the divisor (non-zero)
- dividing a four-digit number by a single-digit number where there is a remainder.

To assess pupils' understanding, provide a selection of division problems and ask them to notice which ones will require regrouping without them having to perform the entire calculation.

Pupils can deepen their understanding by solving problems involving reasoning about the written method such as **Division Rules** from NRICH.

Further resources

- Primary Magazine Issue 61: **Maths to Share - Division**.

You can read more about conceptual and procedural variation **here**.

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Comments

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10 July 2015 09:02
This is a really useful article that I will recommend to many colleagues. It makes perfect sense and the additional scrutiny of numbers looking at what is the same and what has changed makes written methods so much interesting.
By **barbaracarr** **Alert us about this comment**

13 July 2015 14:33
An interesting model, though how do we address the central finding of the Ofsted '20 successful schools' report, that most of the schools found it difficult to teach vertical long division successfully? Since the formal method of long division is an abbreviated representation of efficient grouping/repeated subtractions, it does not directly develop from short division by a sharing model. There is not a parallel development here as we find between methods for short multiplication and long multiplication.

On the other hand short division could be taught as a further abbreviation of the written recording of long division.

Incidentally, the term 'bus shelter' (not 'stop') actually came from an archaic slang used by teachers and pupils in the 1950s and 60s for the layout of the frame for vertical methods of division, it is not a 'method', and is often applied to both short and long division. We need to be careful about which method we actually mean.
By **RalphManning** **Alert us about this comment**

13 July 2015 16:17
Hi Ralph,

The issue of long division is another article I think! I have found the area model that is promoted in the Young Mathematicians at Work series by Fosnot and Dolk **http://www.heinemann.com/products/E00354.aspx** , to be a useful representation of the successive subtraction of multiples of the divisors. This is similar to chunking but enables pupils to work on/ refine their models until they have an efficient strategy for subtracting the largest multiples of hundreds, tens and ones in each step (i.e. the second example in Appendix 1 of the new NC) and thus able to compare what is the same/ different between the visual representation and the abstract written method to deepen conceptual understanding. Googling 'area model for division' will provide you with some video examples of how this is done.

I highly recommend the Young Mathematicians at Work series for deepening your own subject knowledge for PD providers in particularly.
By **Laurie_Jacques** **Alert us about this comment**

14 July 2015 13:03
Thanks Laurie. The area model is indeed a helpful way of representing subtractions and can help as we encourage children to look for increasingly larger and more efficient chunks - a vital but unfortunately neglected aspect of teaching division by subtractions as evidenced in the '20 successful schools' report.

What really concerned me is the disconnect between the two different models and the need therefore to teach the chdn to use both models - one advocated in this article for short division and another (e.g. area model) for long division, when in effect we could use the same (area) model for both forms of vertical written division.

Thanks for the book recommendation.
By **RalphManning** **Alert us about this comment**

14 July 2015 23:08
Hi Ralph,

I think that's a fair point and one that I'd be happy to concede to but maybe there is potential for linking the two representations (counter arrays and the area model) and I will give this some consideration over the summer and maybe you might want to as well?
By **Laurie_Jacques** **Alert us about this comment**

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