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Primary & Early Years Magazine 76

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New National Curriculum in Focus

New National Curriculum in Focus is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study for mathematics. You can find previous features in this series ***here***

Designing learning for short multiplication

The new curriculum requires children to learn to use standard written methods sooner than has been taught in recent years. In their report **Good practice in primary mathematics: evidence from 20 successful schools**, Ofsted identified that in the most successful schools pupils were moved to standard written methods swiftly, and once pupils were secure with interim methods, they were moved quickly on to more efficient methods.

What is important is that, if pupils are to be expected to move to a standard written method more quickly than previously expected, we need to ensure that that they do this not only with procedural fluency but with conceptual understanding.

So can this be achieved for short multiplication?

Let's consider the Y5 statement from the programme of study:

> *multiply numbers up to four digits by a one-digit number … using a formal written method*

Before pupils can begin to learn to do this there are a number of skills and concepts that need to have been developed in order to carry out short and long multiplication:

- Recall fluently multiplication facts
- Visualise and understand how a four-digit number can be partitioned and recombined into multiples of 1000, 100, 10 and 1 with both concrete and abstract representations. (i.e. base 10 (concrete), place value counters or arrow cards)
- Visualise the relative quantity of the numbers
- Know the value of a digit because of its position in a number
- Understand the effect of multiplying by 10, 100 and 1000
- Know and use the fact that multiplication is commutative
- Know and use the fact that multiplication is distributive over addition
- Decide when it is more efficient to calculate mentally.

Scaffolding learning through procedural and conceptual variation

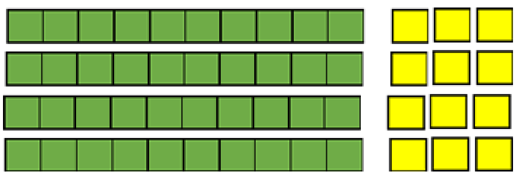
One particular feature of the teaching seen in Shanghai has been the use of teaching with conceptual and procedural variation. You can read in more detail what this means in the National Curriculum in Focus article in **Issue 74**.

So what might conceptual and procedural variation look like in the context of teaching the written method for multiplication of four-digit numbers by a single-digit number? Written multiplication involves finding partial products and adding them together to find the whole product. Pupils will need to recognise that there could be up to four partial products to find the total in a four-digit number multiplied by a single-digit number.

In KS1 and lower KS2 an effective representation of multiplication is to consider this as an array moving to an empty area model.

13 x 4 might be represented using concrete or visual representations below:

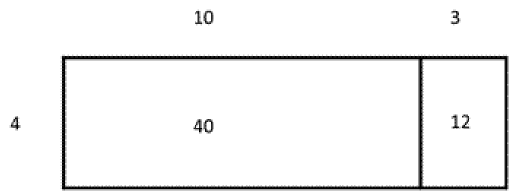
Using concrete representations (Dienes apparatus)



This representation enables pupils to see how multiplication is distributive over addition. E.g. they might see that 13 x 4 is the same as “four tens and four threes” (or another vertical partitioning)

This representation also enables pupils to see how multiplication is associative. E.g.13 x 4 is the same as “two thirteens, twice” (13 x 2) x 2.

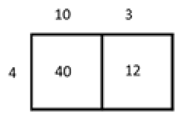
Using an area model



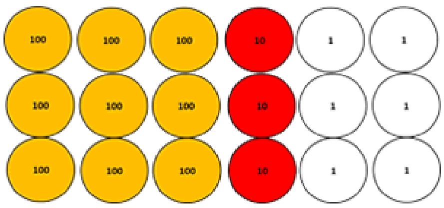
This model provides a more sophisticated and abstract representation of the Dienes representation above and children should be comfortable with the former before moving to this representation.

The area model is then easily translated to the multiplication of larger numbers. However as numbers become bigger (e.g. four-digit) it would become difficult to represent this to scale and therefore the grid representation then becomes an abstraction of this.

E.g.



Place value counters can also be a helpful representation of multiplication - for example, 432 x 3 might be represented in this way with place value counters. It is important to recognise that although the counters are arranged in an array, the 10 counter represents 10 ‘ones’ pushed together in this representation. The 100 counter represents 100 ‘ones’ pushed together.

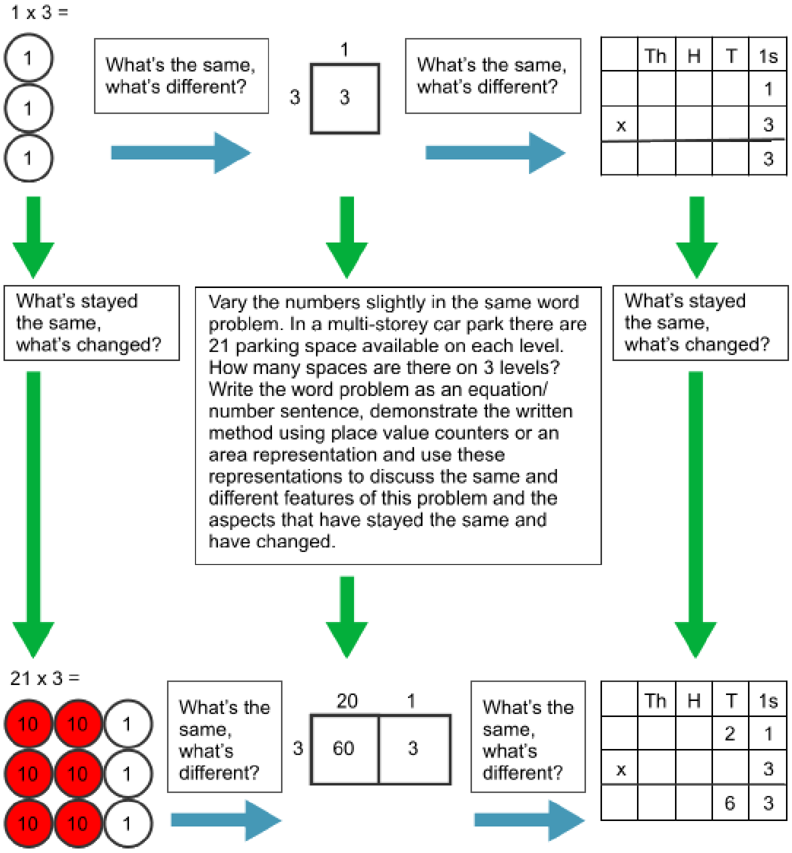


In many primary schools pupils have become confident users of the grid representation to solve multiplication problems. However this method becomes cumbersome as the multiplicand and multipliers increase in size. The number of partial products to add together can mean that children make mental calculation errors. The grid representation can support pupils’ conceptual understanding of multiplication and mental calculation skills but it can also help to move pupils’ towards the standard written method for multiplication (in this case short multiplication).

So how should we help children to learn to become fluent with the written method and retain their conceptual understanding? Below is a sequence of problems that can be used to illustrate how to shift from the grid representation as a jotting for mental calculation to the standard written methods for short multiplication. An assumption is made that the key skills and concepts mentioned above have all been acquired in order for the pupils to work on this process. Also, as for addition and subtraction we also have instances when a number needs to be regrouped. E.g. in 2324 x 2 each successive digit multiplied by 2 will yield a single digit number and no regrouping is required. However in 2326 x 2 we find that the ones digit multiplied by 2 will yield a two-digit number which complicates the written method and requires numbers to be regrouped.

Short multiplication without regrouping

Begin with a simple word problem. Eg. In a multi-storey car park there is one parking space available on each level. How many spaces are there on three levels? Write the word problem as an equation/ number sentence, demonstrate the written method using place value counters or an area representation and use these representations to discuss the same and different features of this problem.



The variations from left to right draw the pupils' attention to the meaning of the increasingly abstract representations. Pupils should be encouraged to seek the similarities for how the partial products are represented in each case. Some children may find the jump from the grid to the written model harder to grasp when there are two (or more) partial products. In which case it might be necessary to add a further interim step of presenting each of the partial products below the solid line and then perform and addition of the partial products. It is important to stress that this is an interim step to ensure conceptual understanding and not a final written method.

E.g.

	Th	H	T	1s
			2	1
x				3
				3

\+

	Th	H	T	1s
			2	1
x				3
			6	0

And/ or moving to…

	Th	H	T	1s
			2	1
		x		3
				3
		+	6	0
			6	3

…before moving on to the standard written method.

Continue with the sequence of problems to build up understanding of the written method without regrouping up to four-digit numbers. Ensuring that the word problem is used as the basis of the numbers at each step.

E.g.

321 x 3
1321 x 3

Pupils can then practise the written method using similar sequences that involve multiplying without regrouping.

E.g.

1312 x 3
1321 x 3
1213 x 3
1231 x 3
1123 x 3

1132 x 3

The above sequence will enable teachers and pupils to focus in on the value of the digits in each product. Because the numbers involved do not require any regrouping the 6 appears in the products sometimes as ones, tens or hundreds and this can lead to a rich discussion about why.

As pupils become more fluent provide a variety of contexts for the problems they are solving using multiplication still without regrouping. Pupils should also be given the opportunity to explore numbers where the multiplicand contains a 0 in various places and notice the number of non-zero partial products there are to add together in each case.

Short multiplication with regrouping

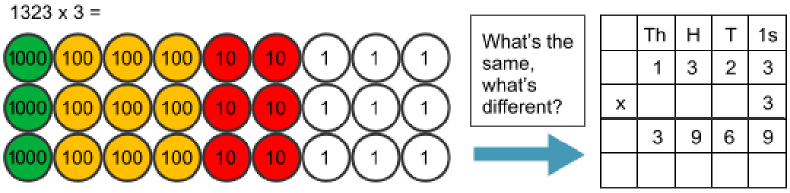
Provide a sequence of problems using a familiar context that will reveal a change in the process that leads to pupils becoming aware of the need to regroup. The point of the exercises will be to help the pupils to generalise when there is a need to regroup. The pivot points are the points at which we want the children to notice something that changes.

E.g.

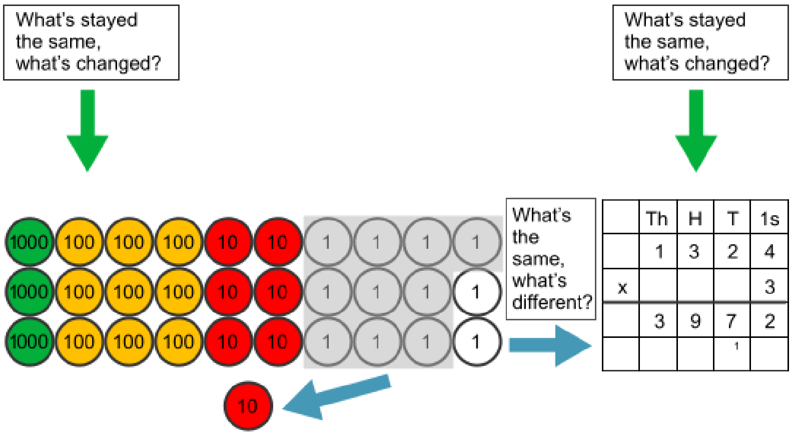
1322 x 3
1323 x 3
1324 x 3 (pivot point 1)
1325 x 3
1326 x 3
1327 x 3 (pivot point 2)

Pivot point 1 in the above is the first occurrence of a procedural change that the sequence will enable the pupils to observe. The pupils will observe that 'ten ones' has been regrouped into 'one ten'. See below for a representation using place value counters.

1323 x 3 =



1324 x 3 =



Pivot point 2 involves a further procedural change as pupils will need to observe that 'twenty ones' will need to be regrouped into 'two tens'. Pupils will then need the opportunity to practise with other sequences, observing and applying regrouping ones in to tens accordingly. Pupils will be secure with this process when they understand and articulate a generalisation that describes when regrouping of ones will occur. Pupils could be provided with a selection of multiplications and asked to notice which ones will require regrouping without them having to perform the entire calculation.

Pupils may then be moved to sequences that enable them to observe that the tens need to be regrouped into hundreds or hundreds into thousands and examples where regrouping may occur more than once in a calculation. These are other 'pivot points' to design for learning to use the written method for short multiplication. Try to think of some other examples and share the sequences in the comments box below. The starting point for a sequence of problems will depend on the children's conceptual understanding and procedural fluency of previous written methods. E.g the children may be fluent in their use of multiplying three-digit numbers by a single-digit number, in which case it may be appropriate to begin with a three-digit number and vary the thousands number. The examples above are illustrative. You may wish to use different representations or further interim steps to ensure pupils develop conceptual understanding with procedural fluency for the written method. However a key point to consider is that any interim method is best used for short periods only. They are conceptual stepping stones to something more efficient. If children stay on these methods for any length of time, they may become fixed as procedures, making it difficult to move on. The conceptual models might be revisited later to link other mathematics. The grid method can be valuable at Key Stages 3 and 4 within the context of algebra.

Pupils can deepen their understanding by solving problems involving reasoning about the written method such as **All the Digits** from NRICH.

Or find the hidden digits

	Th	H	T	1s
	1	★	4	5
x				5
	6	7	★	★

Or how many ways can you find to make this calculation work?

	Th	H	T	1s
	1	★	4	5
x				5
	★	7	★	★

Further resources:

- Primary Magazine Issue 60: **Maths to Share - Multiplication**
- You can read more about conceptual and procedural variation **here**.

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17 June 2015 06:36
After staff CPD I see the potential for these images to also be used for a calculation policy. Images and annotations show the learning journey to an efficient written method.
By **lisaholt** **Alert us about this comment**

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