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Primary & Early Years Magazine 74

New National Curriculum in Focus

New National Curriculum in Focus is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study for mathematics. You can find previous features in this series ***here***

Designing learning for column addition

The new curriculum requires children to learn to use standard written methods sooner than has been taught in recent years. In their report **Good practice in primary mathematics: evidence from 20 successful schools**, Ofsted identified that in the most successful schools pupils were moved to standard written methods swiftly and once pupils were secure with interim methods were moved quickly on to more efficient methods.

What is important is that, if pupils are to be expected to move to a standard written method more quickly than previously expected, we need to ensure that that they do this not only with procedural fluency but with conceptual understanding.

So can this be achieved for the written method of addition?

Let's consider the Y3 statement from the programme of study:

> *add … numbers with up to three digits, using formal written methods of columnar addition*

Before pupils can begin to learn to do this there are a number of skills and concepts that need to have been developed in order to carry out column addition with conceptual understanding:

- Visualise and understand how a three-digit number can be partitioned and recombined into multiples of 100, 10 and 1 with both concrete and abstract representations (i.e. base 10 (concrete) or arrow cards)
- Visualise the relative quantity of the numbers.
- Know the value of a digit because of its position in a number
- Know that addition is commutative
- Be able to say that a three-digit number is greater than *a* but less than *b*
- Be able to mentally add:
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds.

Scaffolding learning through procedural and conceptual variation

One particular feature of the teaching seen in Shanghai has been the use of teaching with conceptual and procedural variation. Teaching with conceptual variation involves the comparison of static models and images of a mathematical concept which enables pupils to compare by identifying things that are the same and different about the representations which then help to reveal the essential and non-essential features of a mathematical concept. For example multiple examples of different triangles will enable pupils to generalise that for a shape to be a triangle it has to be a closed shape with three straight sides and three vertices (i.e. the essential features of a triangle). The non-essential features being side length, angle size, and orientation with respect to a horizontal line. Teaching with procedural variation relates directly to Bruner and Wood's ***Scaffolding*** (1976) and involves teaching a mathematical process in such a way that the process is gradually 'unfolded' through a succession of carefully chosen steps so as to gradually enable the child to determine 'what stays the same', and 'what changes' in each successive step. This enables the pupils to identify the variant and invariant features of the process, seeing connections between steps and leads to a generalisation that can be applied to all situations when the process is used.

So what might conceptual and procedural variation look like in the context of teaching written addition of three-digit numbers? Written addition involves two distinct processes: adding without regrouping and adding with regrouping. Regrouping occurs when, for example, the sum of the two numbers in the 1s place is greater than 9. We begin without regrouping.

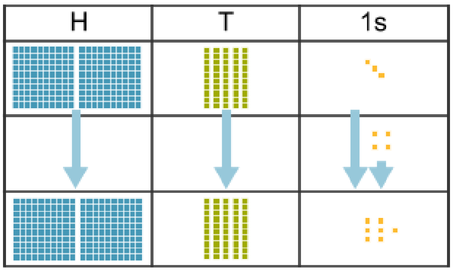
An assumption is made that the key skills and concepts mentioned above have all been acquired by the pupils working on this process.

Begin with a simple word problem. Eg. *A sheep farmer has 253 ewes and 4 rams. How many* *sheep are there altogether?* Write the word problem as an equation/ number sentence, demonstrate the written method using the base 10 resources and use these representations to discuss the same and different features of this problem:

What's the same, what's different?

253 + 4 =

H	T	1s
2	5	3
		4
2	5	7



What's stayed the same, what's changed?

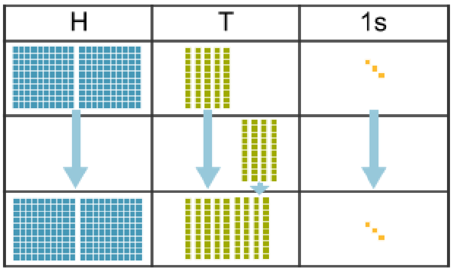
What's stayed the same, what's changed?

Vary the numbers in the same word problem. Eg. *A sheep farmer has 253 ewes and 40 rams. How many sheep are there altogether?* Use this word problem as before to write the equation/ number sentence, demonstrate the written method with the base 10 resources and use these representations to focus on the similarities and differences of the representations for this problem and the aspects that have and have not changed between this and the previous problem:

What's the same, what's different?

253 + 40 =

H	T	1s
2	5	3
	4	0
2	9	3



What's stayed the same, what's changed?

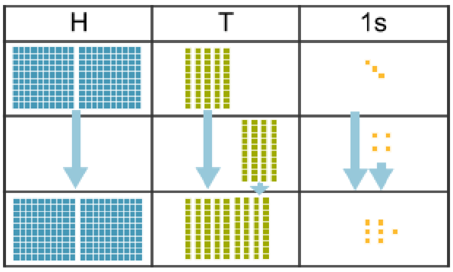
What's stayed the same, what's changed?

Vary the numbers in the same word problem. Eg. *A sheep farmer has 253 ewes and 44 rams. How many sheep are there altogether?* Use this word problem as before to write the equation/ number sentence, demonstrate the written method with the base 10 resources and use these representations to focus on the similarities and differences of the representations for this problem and the aspects that have and have not changed between this and the previous problem:

What's the same, what's different?

253 + 44 =

H	T	1s
2	5	3
	4	4
2	9	7



What's stayed the same, what's changed?

What's stayed the same, what's changed?

The variations from left to right draw the pupils' attention to the concrete representation which helps to reinforce the place value of the digits as well as providing an image for the vocabulary when a sentence such as "we add the five tens and four tens".

The variation in successive steps helps to draw the pupils' attention to what changes when ones or tens are added together.

This sequence could then continue with the pupils working with the teacher on a few further successions. Such as:

253 + 45 =
253 + 46 =
253 + 34 =
253 + 24 =

Before pupils work independently on a task such as:

Use the digits 2 5 3 4 1 to make a three-digit number and a two-digit number.
What's the largest total you can make? What's the smallest?

Learning can be deepened by offering further questions such as…

- What's the largest even total you can make? What's the smallest?
- What's the largest odd total you can make? What's the smallest?
- A total nearest to 500? 300? Etc.
- Arrange five digits into a three-digit and two-digit number which sum to a given total.

The examples above are also easily solvable by mental calculation and this could form part of the discussion towards the end of the lesson about how the children might now solve this problem in their head. Some pupils might refer to a number line, counting on or visualising the method taught above.

A subsequent lesson might build on this work by solving similar structured word problems with three-digit and two-digit numbers, adding a three-digit number to more than two two-digit numbers without regrouping or adding two three-digit numbers (without regrouping) using a similar sequence above but with hundreds, tens and 1s.

Addition with regrouping

Use a familiar word problem. Eg. *A sheep farmer has 253 ewes and 6 rams. How many sheep are there altogether?* and continue to use this as the basis for the sequence of problems. Regrouping may take longer to learn conceptually.

The sequence might develop (using the base 10 resources) in this way. 'What's the same and what's different?' and 'What's stayed the same and what's changed?' will, as before, support the successive steps.

Working together with the teacher:

253 + 6 =
253 + 7 = (this problem is a pivot point for introducing the new concept)
253 + 8 =

Working independently:

257 + 2 =
257 + 3 =
257 + 4 =
257 + 5 =
235 + 4 =
235 + 5 =

Using variation for regrouping will enable pupils to observe when regrouping of 1s into tens will need to happen. Sequencing the successive steps carefully will enable pupils to draw a generalisation about the sum of the 1s being greater than nine.

Using conceptual and procedural variation can appear to be slowing down learning but taking time when introducing a new concept or process in this way can develop deep understanding when the concept or procedure is new and thus avoids pupils repeating identical lessons year on year because they haven't understood.

Further resources:

- Primary Magazine Issue 58 **Maths to Share: Addition**.

You can read more about conceptual and procedural variation **here**.

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20 April 2015 11:56
I found it helpful to ask the children to write a simple addition next to the column layout, once they were ready for this method. For example, if there were 3 units and 4 units, they would write 3+4=7 by the side, then AFTER the units, they add the tens as 20+30=50. We then discussed whether they should write 50 or 5 in the tens column to reinforce the place value knowledge. Also used this for subtraction in columns, not using decomposition of course.
By **cwearden** **Alert us about this comment**

05 May 2015 14:28
The next edition will focus on subtraction.
By **Laurie_Jacques** **Alert us about this comment**

14 September 2015 10:26
You have not mentioned about pupils using the expanded method of addition as a useful building block towards conceptual understanding.234 +176 would be challenging, yet using

base ten, a template for HTO and patitioning, my pupils were able to successfully work out the answer.

200+30+4

!00+70+6

Later, they appeared much more confident when beginning the formal columnar method.

It is an additional step to include, but one that reaped benefits for work later on. I did the expanded form one term and then the normal columnar method the next and by the end of the year my Year Three children were successfully using and understanding the formal method.

By **joansmith** **Alert us about this comment**

14 September 2015 12:45

The expanded methods may have their place. But Ofsted have seen many schools teaching it as a written method rather than as an interim step. The expanded method maybe helpful for children struggling to make sense of what is happening but I think it's important to discuss and decide as a school whether a focus on the use of manipulatives in place of expanded methods is a more efficient learning journey for pupils who can become bogged down in all the additional recording that they require.

it isn't about the length of time you move on it's about whether the pupils are ready to and some maybe being held back by expanded methods being taught when they are not needed.

By **Laurie_Jacques** **Alert us about this comment**

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